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Harmonic Perturbations in Turbulent Wakes

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A theoretical model of harmonic perturbations in far turbulent wakes is considered. The proposed model is based on the triple decomposition method. It is assumed that the instantaneous velocities and pressures consist of three distinctive components: the mean (time-averaged), the coherent (phase-averaged), and the random (turbulent) motion. The interaction between incoherent turbulent fluctuations and large-scale coherent disturbances is incorporated by means of a Newtonian eddy viscosity model. For high-amplitude perturbations, the nonlinear feedback to the mean flow is taken into account by means of the coherent Reynolds stresses. The equations for the mean flow are coupled with the linearized equations for the disturbances, taking into account the mean flow nonparallel effects. The model resolves uncertainties noted in previous theories and provides a correct comparison with available experimental data. The effect of the harmonic perturbations on the turbulent wake growth at high amplitudes is investigated as well.

Nomenclature

$\boldsymbol{A}$	=	vector function of the direct problem
$a(X)$	=	slow amplitude function of the coherent perturbation
$\boldsymbol{B}$	=	vector function of the adjoint problem
f	=	frequency, Hz
G	=	growth parameter, $L_0 U_\infty/\theta u_0$
H_1, H_{10}, H_{11}, H_2, H_3	=	matrices 4×4
L_0	=	half-width of the wake
p	=	instantaneous pressure
Re	=	Reynolds number, $U_\infty \theta/\nu_t$
t	=	time
U	=	mean (time-averaged) streamwise velocity
U'	=	$\partial U/\partial y$
U_∞	=	freestream velocity
u	=	instantaneous streamwise velocity
u_0	=	centerline velocity deficit
V	=	mean (time-averaged) vertical velocity component
V_0	=	$\varepsilon^{-1} V$
v	=	instantaneous vertical velocity
X	=	slow variable, εx
x	=	streamwise coordinate
$\bar{x}$	=	dimensionless coordinate, $x/2\theta$
y	=	vertical coordinate
z	=	spanwise coordinate
α	=	wave number of the coherent perturbation
ε	=	small parameter characterizing the flow divergence
ε_0	=	amplitude parameter, $\tilde{v}_{\max}/U_\infty$
η	=	dimensionless coordinate, y/L_0
θ	=	momentum thickness
ν_t	=	eddy viscosity of the undisturbed flow
ω	=	frequency of the coherent perturbation
$\tilde{}$	=	coherent (phase-averaged) contribution
$'$	=	random contribution
$\langle\,\rangle$	=	phase averaging
$-$	=	time averaging
$*$	=	Hermitian adjoint matrix

Subscripts

i, j	=	coordinates x, y
max	=	maximum value of the coherent component at specific coordinate x

Introduction

STUDIES of large organized motions in turbulent flows have a long history and demonstrate that large-scale vortices originate from flow instabilities and that excitation of time-periodic large coherent structures in turbulent shear flows may serve as a method of flow control. Flow instabilities may cause small disturbances to be amplified and, in turn, provide feedback to the basic flow. A detailed discussion of this phenomena and a related bibliography are given by Ho and Huerre.[1]

Experiments with harmonic perturbations introduced into turbulent mixing layers and wakes revealed that the flows are highly susceptible to the disturbances.[2–8] The link between the perturbation dynamics and stability theory has been demonstrated by Gaster et al.[3] (mixing layer), Wygnanski et al.,[4] Cimbala et al.,[8] and Marasli et al.[6] (far wake flow).

The triple decomposition method, where the instantaneous velocities and pressure are considered as sums of three distinctive components, mean (time-averaged), coherent (phase-averaged), and random (incoherent turbulent) motion, is an appropriate method for analyzing a coherent signal in a turbulent flow. The equations for a coherent signal in a turbulent flow were derived by Reynolds and Hussain[9] and contain new unknown terms that correspond to oscillations of the background Reynolds stresses caused by the organized wave. These terms reflect the existence of an interaction between the coherent signal and the random field. As a first approximation, the interaction may be neglected, and the linearized problem may be reduced to the Rayleigh equation with a prescribed mean velocity profile (the effect of molecular viscosity is considered as negligible).

Inasmuch as free shear flows (such as jets, wakes, and mixing layers) spread rapidly, it is necessary to take into account the flow

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divergence for a correct comparison of theoretical and experimental data for growth rates and overall amplification. Gaster et al.[3] carried out an analysis of coherent perturbations in a mixing layer within the scope of the Rayleigh equation. They included flow divergence in an attempt to predict the overall growth. Nevertheless, the theory overpredicted the amplification by two- to threefold.

Marasli et al.[6] emulated the interaction between the coherent perturbation and the turbulent wake field via the introduction of eddy viscosity and consideration of the Orr–Summerfield equation for the coherent signal. Flow divergence effects were taken into account as well. The analysis in both the aforementioned theoretical models was based on the mean flow velocity profile that was obtained as an approximation to the experimental data.

It follows from the experiments that the nonlinear effect of the feedback from the disturbances to the mean flow might be very significant, and a self-consistent theory has to take it into account. Cohen et al.[10] considered the feedback from harmonic perturbations to the mean flow of a mixing layer. The theoretical model was based on a so-called quasi-parallel approach. The Reynolds stress of the coherent signal was calculated using a solution of the Rayleigh equation, and the disturbance amplitude was calculated in accordance with the growth rate obtained from the Rayleigh equation. Because the self-similar velocity profile was obtained as an approximation to the experimental data,[3] it was necessary to find only the mixing layer thickness as a function of the downstream coordinate. The equation for the mixing layer scale follows from the integral momentum equation, where the spreading rate is governed by the coherent Reynolds stresses. The results[10] showed an agreement between the theoretical and experimental data.[3]

Marasli et al.[11] applied the same approach to the analysis of the feedback from a coherent signal to the mean flow of a turbulent wake. The main difference involved taking into consideration the interaction of the coherent signal and turbulence by means of an eddy viscosity model. The theoretical results[11] also showed an agreement with experimental data.[6] However, one should keep in mind that the quasi-parallel approach suffers from an ambiguity of the linear solution normalization. The solution of the Orr–Sommerfeld equation (and the Rayleigh equation) depends on the downstream coordinate as a parameter, and an undetermined complex function of the downstream coordinate appears. It must be defined in some way or another. The normalization of the eigenfunction influences the resulting level of the Reynolds stresses, as well as the final comparison of the theory with the experimental data. The normalization adopted in the aforementioned papers[10,11] was not stated. According to J. Cohen and V. Levinski (private communication, 1997), the value of the disturbance stream function at the centerline was used to normalize the eigenfunctions of the linear problem.

Because the divergence of the mean flow is taken into account, the uncertainty in the eigenfunction normalization becomes resolved. Reau and Tumin[12] proposed a model for the analysis of a coherent signal in a turbulent mixing layer. A Newtonian eddy viscosity model was used to incorporate the interaction between incoherent turbulent fluctuations and large-scale coherent disturbances. For high-amplitude perturbations, the nonlinear feedback to the mean flow was taken into account by means of the coherent Reynolds stresses, and the flow divergence was taken into account as well. The result demonstrated agreement with available experimental data and revealed the possibility of a negative spreading rate of the mixing layer, as observed in the experiments of Weisbrot and Wygnanski.[5]

The objective of the present work is to apply the proposed method[12] to the analysis of a coherent signal in far turbulent wakes.

Governing Equations

It is assumed that the instantaneous value of a parameter $q(x, y, z, t)$ (velocities and pressure) consists of the three distinct components

$$q(x, y, z, t) = Q(x, y) + \tilde{q}(x, y, t) + q'(x, y, z, t) \tag{1}$$

where Q is the mean (time-averaged) value, $\tilde{q}$ is the coherent (phase-averaged) contribution of the organized wave, and q' stands for the random (turbulent) motion. After substituting Eq. (1) into the Navier–Stokes equations and phase and time averaging, the equations for the mean flow and the organized wave can be obtained.[9]

We assume that the effects of molecular viscosity are negligible. The boundary-layer approximation is assumed for the mean-flow equation, and we invoke closure by means of the eddy viscosity ν_t:

$$\overline{u'v'} = -\nu_t \frac{\partial U}{\partial y} \tag{2}$$

where the overbar stands for the mean (time-averaged) value.

The mean-flow equations are written in conventional notation as follows:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{3a}$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{\partial \overline{\tilde{u}\tilde{v}}}{\partial y} - \frac{\partial \overline{\tilde{u}^2}}{\partial x} + \nu_t \frac{\partial^2 U}{\partial y^2} \tag{3b}$$

Equation (3b) contains coherent Reynolds stresses and, therefore, there is a possibility for feedback from the coherent signal to the mean flow. To solve Eqs. (3a) and (3b), we need to define a velocity profile $U(x_0, y)$ at $x = x_0$ and boundary conditions at $y \to \pm\infty (U \to U_\infty)$. Although we consider a small-deficit far wake, we do not linearize the momentum equation because we want to utilize the solver developed for the mixing layer problem in Ref. 12.

The equations for the organized waves contain the new terms[9]

$$\tilde{r}_{ij} = \langle u_i' u_j' \rangle - \overline{u_i' u_j'} \tag{4}$$

The new terms represent oscillations of the incoherent Reynolds stresses caused by the organized wave. They are responsible for the interaction of a coherent signal with the random field. Because of the latter terms, we need to invoke an hypothesis to close the equations. If we assume that $\tilde{r}_{ij} = 0$, the linearized equations for organized waves are reduced to the Rayleigh equation. According to Ref. 9, we adopt the Newtonian eddy viscosity model

$$\tilde{r}_{ij} = -\nu_t\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) \tag{5}$$

Making use of Eq. (5), we obtain equations for a coherent signal. In linearized form, they are written as follows:

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{6a}$$

$$\frac{\partial \tilde{u}}{\partial t} + U\frac{\partial \tilde{u}}{\partial x} + V\frac{\partial \tilde{u}}{\partial y} + \tilde{u}\frac{\partial U}{\partial x} + \tilde{v}\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial x} + \nu_t\left(\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2}\right) \tag{6b}$$

$$\frac{\partial \tilde{v}}{\partial t} + U\frac{\partial \tilde{v}}{\partial x} + V\frac{\partial \tilde{v}}{\partial y} + \tilde{u}\frac{\partial V}{\partial x} + \tilde{v}\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial y} + \nu_t\left(\frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial y^2}\right) \tag{6c}$$

Equations (6a–6c) should be completed by the boundary conditions at $y \to \pm\infty$ $(\tilde{u}, \tilde{v}, \tilde{p} \to 0)$.

Linear Analysis: Slowly Diverging Flow

In what follows, we use the momentum thickness θ as the length scale and U_∞ as the velocity scale. We introduce the vector function

$$\boldsymbol{A} = \left|\begin{matrix} \tilde{u} \\ \tilde{p} \\ \tilde{v} \\ \dfrac{\partial \tilde{u}}{\partial y} - \dfrac{\partial \tilde{v}}{\partial x} \end{matrix}\right| \tag{7}$$

and rewrite Eq. (6) in the following form:

$$\frac{\partial \boldsymbol{A}}{\partial y} + H_{10}\frac{\partial \boldsymbol{A}}{\partial t} = H_{11}\boldsymbol{A} + H_2\frac{\partial \boldsymbol{A}}{\partial x} + \varepsilon H_3 \boldsymbol{A} \tag{8}$$

where the matrix H_3 is associated with the terms that originated from a slow divergence of the mean flow,

$$H_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -Re & 0 & 0 & 0 \end{pmatrix}, \qquad H_{11} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & U'Re & 0 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -U & -Re^{-1} \\ -1 & 0 & 0 & 0 \\ URe & Re & 0 & 0 \end{pmatrix}$$

$$H_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ V_0\dfrac{\partial}{\partial x} + \mathcal{O}(\varepsilon) & 0 & -\dfrac{\partial V_0}{\partial y} & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{\partial U}{\partial X}Re & 0 & ReV_0\dfrac{\partial}{\partial x} & V_0 Re \end{pmatrix} \tag{9}$$

where a slow variable $X = \varepsilon x$ appears in H_3, component $V_0 = \varepsilon^{-1} V$, and velocity $U = U(X, y)$.

We are looking for a solution of the form

$$A(x, X, y, t) = \exp\left[i \int_{x_0}^{x} \alpha \, dx - i\omega t \right] \{a(X)A_1(X, y) + \varepsilon A_2(X, y) + \cdots\} \tag{10}$$

To leading orders of ε, we obtain the conventional system of linear stability theory:

$$\frac{\partial A_1}{\partial y} = H_1 A_1 + i\alpha H_2 A_1 \tag{11}$$

where $H_1 = H_{11} + i\omega H_{10}$. We must complete the equations with the boundary conditions of the decaying solution at $y \to \pm\infty$. The system (11) may be reduced to the Orr–Sommerfeld equation for the third component of the vector A_1. The coefficients in Eq. (11) are slow functions of the variable X, and the problem of consistent normalization for the quasi-parallel solution A_1 arises. The ambiguity is resolved through consideration of the next-order equations.

In the next order, we have an inhomogeneous system:

$$\frac{\partial A_2}{\partial y} - H_1 A_2 - i\alpha H_2 A_2 = \frac{da}{dX} H_2 A_1 + a H_2 \frac{\partial A_1}{\partial X} + a H_3 A_1 \tag{12}$$

The solvability condition of the system (12) will be the orthogonality of the right-hand side to the solution B of the adjoint problem:

$$-\frac{\partial B}{\partial y} = H_1^* B - i\bar{\alpha} H_2^* B \tag{13}$$

with the boundary conditions of the decaying solution at $y \to \pm\infty$. The overbar in Eq. (13) stands for the complex conjugate. We write the solvability condition for Eq. (12) in the following form:

$$\frac{da}{dX} = -a\frac{N(X)}{M(X)}, \qquad M(X) = \int_{-\infty}^{+\infty} (H_2 A_1, B)\, dy$$

$$N(X) = \int_{-\infty}^{+\infty} \left(H_2 \frac{\partial A_1}{\partial X}, B \right) dy + \int_{-\infty}^{+\infty} (H_3 A_1, B)\, dy \tag{14}$$

One can show that $a(X)A_1(X, y)$ is independent of a specific normalization of $A_1(X, y)$ at different locations of X. This is one reason why we must take the flow divergence into account. The system of equations (3), (11), and (14) was solved numerically.

Results and Discussion

In this section we present comparisons of experimental data[6,7,11] with the proposed theoretical model. For all cases, the initial velocity profile and the disturbance amplitude are defined at $\bar{x} = x/2\theta = 60$. The Reynolds number Re, based on the momentum thickness θ, the freestream velocity U_∞, and the eddy viscosity, is equal to 31. The eddy viscosity ν_t is estimated from the experimental data as 5×10^{-4} m^2/s (see Ref. 6).

Measurements of the unforced flow[4] showed that, far from a wake generator, the mean flow reaches its self-preserving state with a velocity profile of the form (for more physical details of the self-preserving state see Refs. 13–15)

$$U(x, \eta) = U_\infty - u_0(x)h(\eta) \tag{15}$$

The following approximation of the function $h(\eta)$ has been suggested[6]:

$$h(\eta) = \mathrm{sech}^2(0.78\eta + 0.101\eta^3)$$

It follows from the experiments that the velocity deficit and the wake half-width are described by the equations

$$(U_\infty/u_0)^2 = \left(2/W_0^2\bar{x}\right), \qquad (L_0/\theta)^2 = \left(2\Delta_0^2\bar{x}\right)$$

where W_0 and Δ_0 are constants. For the wake behind a flat plate, $W_0 = 1.676$ and $\Delta_0 = 0.2995$. The velocity profile (15) was used as the initial data in the solution of Eqs. (3a) and (3b).

Results for the unforced case and their comparisons with the approximation of the experimental data are given in Fig. 1 (the numerical methods are described in the Appendix). As shown, the theoretical mean velocity profile is self-similar, as observed in the experiments. The theory slightly underpredicts the coefficient W_0 and overpredicts Δ_0.

Marasli et al.[6] introduced the growth parameter G given by

$$G = L_0 U_\infty/\theta u_0 = (2\Delta_0/W_0)\bar{x}$$

which characterizes spreading of the mean flow. Figure 1 also shows the theoretical growth of the parameter G. The slight discrepancy between theoretical results and the approximated experimental data might be caused by the effect of the initial conditions, which were defined by Eq. (15) and are not accurate at $\bar{x} = 60$.

In the following analysis of perturbations, we restrict ourselves to the sinuous mode only. Figure 2 shows the results for the wake subjected to low-amplitude, $\varepsilon_0 = 1\%$, disturbances at a frequency of 40 Hz. We define the amplitude of the disturbance as $\varepsilon_0 = \tilde{v}_{max}/U_\infty$. (Note that in the experiments[6,7,11] it was defined as $\tilde{u}_{max}/u_0$.) One can see that the effect of the coherent signal on the growth parameter

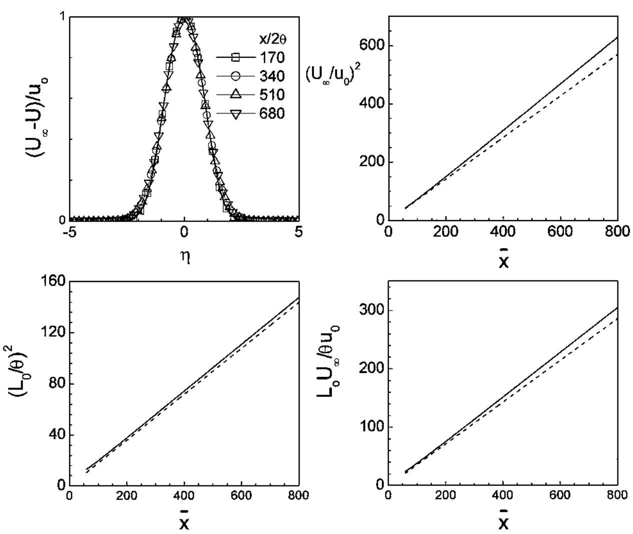

Fig. 1 Unforced case, streamwise distribution of the wake parameters: solid line theory and dashed line approximation of the experimental data.

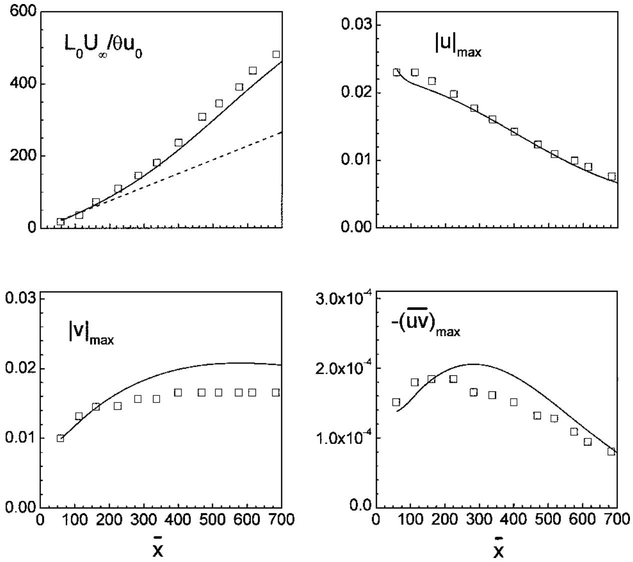

Fig. 2 Theoretical (——) and experimental (symbols) data; dashed line undisturbed flow: f = 40 Hz and ε_0 = 1%.

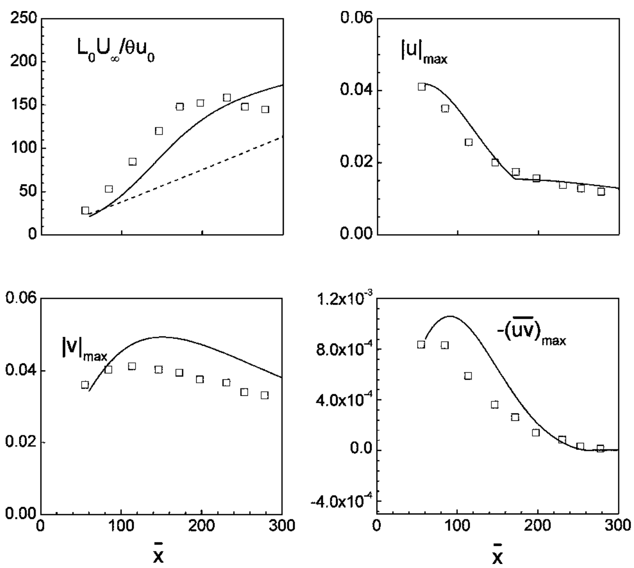

Fig. 3 Theoretical (——) and experimental (symbols) data; dashed line undisturbed flow: f = 80 Hz and ε_0 = 3.43%.

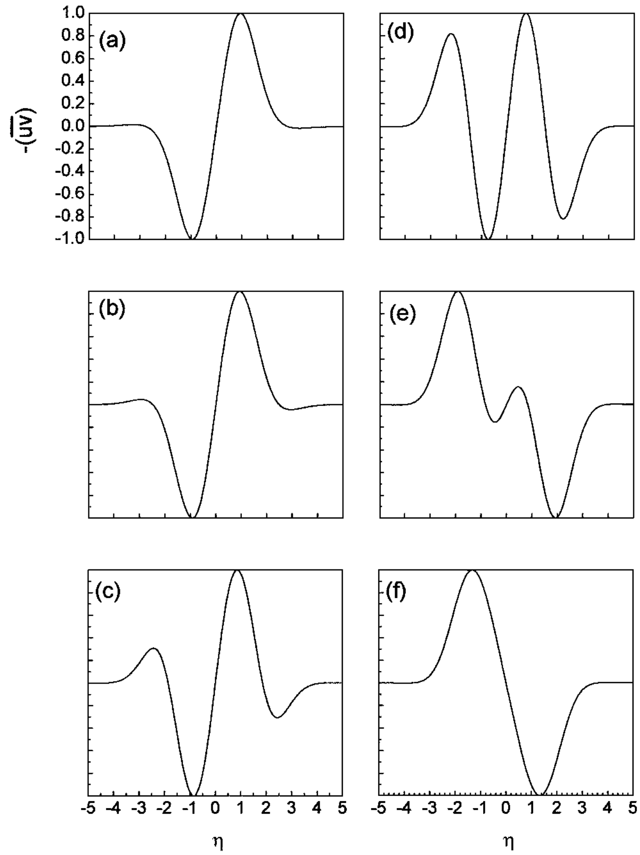

Fig. 4 Forced case, Reynolds stress distributions; f = 80 Hz, ε_0 = 3.43%: a) $\bar{x}$ = 60, b) $\bar{x}$ = 128, c) $\bar{x}$ = 207, d) $\bar{x}$ = 229, e) $\bar{x}$ = 252, and f) $\bar{x}$ = 298.

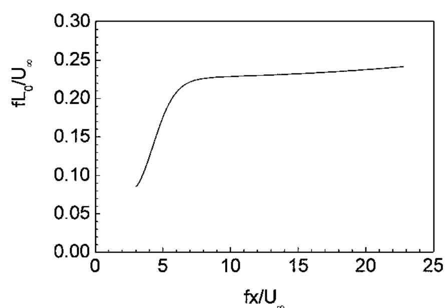

Fig. 5 Variation of the local Strouhal number vs scaled streamwise coordinate: f = 80 Hz and ε_0 = 3.43%.

is significant and that there is good agreement between the theoretical and experimental data. The calculations also revealed that the mean velocity profile remains self-similar. Figure 2 of the present paper corresponds to Fig. 4 in Ref. 11, and both of them demonstrate agreement with the experimental data. The latter indicates that the specific normalization of eigenfunctions used in Ref. 11 leads to $N(X) \approx 0$ in Eq. (14), and both theories could provide close results. In the general case, the solution of Eq. (14) for the amplitude function $a(X)$ strongly depends on the eigenfunction normalization, and the effects of nonparallel flow must be taken into account.

Comparisons with experimental data for the wake in the presence of a coherent signal of $\varepsilon_0 = 3.43\%$ and $f = 80$ Hz are shown in Fig. 3 (corresponds to Fig. 7 in Ref. 11). The agreement between theoretical and experimental data is relatively poor for the growth parameter. This might be attributed to the closure (2) for the incoherent Reynolds stresses. It follows from the experimental data (Fig. 2 in Ref. 11) that the latter cannot be presented as the mean velocity derivative with a constant coefficient ν_t far downstream, past the neutral point of the coherent disturbance.

The effect of the coherent signal on the wake is discussed in Ref. 7. It depends on the behavior of the coherent Reynolds stresses (Fig. 4). At the beginning of the development, the Reynolds stress of the coherent signal is similar to the incoherent one [Eq. (2)] and augments the flow spreading. Marasli et al.[7] pointed out that initially the rate of growth is nearly linear along x; then a break in the slope occurs. The local Strouhal number fL_0/U_∞ at this point is about 0.22. This value is close to the neutral point for the coherent disturbance. Behind the neutral point, the coherent Reynolds stress changes its sign, and the slope of the growth parameter G becomes smaller than in the unforced wake. As the coherent disturbance decays farther downstream, the effect of the coherent Reynolds stress is reduced, the turbulent diffusion process regains its dominance, and the growth parameter starts increasing again.

The variation of the local Strouhal number vs the scaled coordinate fx/U_∞ is shown in Fig. 5. The calculations give the neutral point at $\bar{x} \approx 150(fx/U_\infty \approx 7.5)$, and the local Strouhal number is about 0.22, as observed in the experiments.[7]

The effect of the forcing amplitude on the growth parameter is given in Fig. 6.

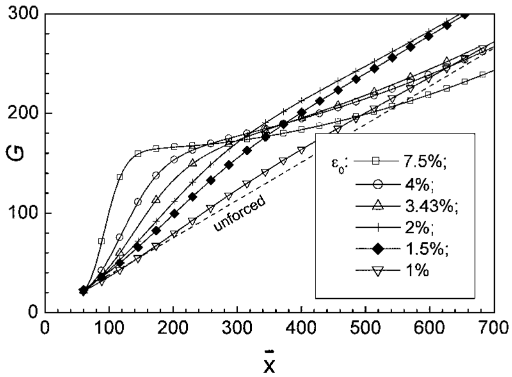

Fig. 6 Calculated effect of the disturbance amplitude on the growth parameter: *f* = 80 Hz.

Conclusions

The triple decomposition method provides an adequate approach to analysis of the experimental data, where the coherent signal is extracted from the total signal by means of phase-locked measurements. The governing equations (3) for the coherent disturbances spell out that the interaction between the coherent and random components might be important, and it should be treated somehow. A closure hypothesis for these equations should be adopted, and it can be checked only by comparison with experimental data.

The successful comparison of the present theoretical results and the experimental data related to the mean flow and to the coherent signal leads to the following conclusions:

1) The Newtonian eddy viscosity model for the coherent signal provides the quantitative agreement with the experimental data.

2) Interaction between the coherent signal and the random field is important.

3) The flow divergence must be taken into account to provide a correct comparison of theoretical and experimental data.

4) The feedback of the disturbances to the mean flow is significant, and it can be taken into account via coherent Reynolds stresses.

Appendix: Numerical Methods

The numerical procedure to treat Eqs. (3a) and (3b) was based on the finite difference scheme used for the boundary-layer equations.[16–18] We solve Eqs. (3a) and (3b) separately for the upper ($y > 0$) and lower ($y < 0$) domains. At the boundary $y = 0$, the velocity $U(x, 0)$ is iterated to obtain the coincidence of the derivatives $\partial U/\partial y$ for the upper and lower domains. The code was tested with available result for mixing layers and wakes without the feedback term in Eq. (3b) associated with the coherent Reynolds stresses.

For the stability analysis of the mixing layer we used three codes:

1) Code 1 is based on the two-domain spectral collocation method with Chebyshev polynomials (see Ref. 19) at finite Reynolds number.

2) Code 2 has the fourth-order Runge–Kutta scheme at finite Reynolds number when two pairs of the decaying fundamental solutions are calculated for the upper and lower domains. The general solution is obtained as a sum of the two fundamental solutions for each domain, and an orthonormalization procedure is used during calculation for each pair of fundamental solutions.

3) Code 3 has the fourth Runge–Kutta scheme for the Rayleigh equation.

The code with the spectral collocation method provides a map of eigenvalues α for a prescribed frequency and Reynolds number. The eigenvalue problem is reduced to a generalized matrix eigenvalue problem of the form $AX = \alpha BX$ where A and B are the square matrices and X is an eigenvector. The problem was solved by means of a standard routine from the Numerical Algorithms Group or International Mathematical and Statistical Library FORTRAN libraries. The method was realized with 80 Chebyshev polynomials in two domains: $-200 \leq y \leq 0$ and $0 \geq y \geq 200$. Because the Reynolds number is sufficiently high, the code provides an initial eigenvalue for the Rayleigh equation. If the Reynolds number is finite, the code provides an initial eigenvalue for code 2. Comparisons of the eigenvalues obtained from these three stability codes (up to 4–5 figures in the eigenvalues) served as verification of the codes.

In addition, two codes were prepared for the analysis of the slow diverging phenomenon:

1) This code is based on the inviscid equations similar to Ref. 3.

2) This code is based on the equations with a finite Reynolds number as described in this paper.

Repeating the results from Ref. 3 for the mixing layer flow tested the first code. Afterward, these results were obtained with the second code, when the Reynolds number was chosen to be sufficiently high. Finally, the second code was used for the analysis at finite Reynolds numbers.

Acknowledgments

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